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were balloted for and duly elected Fellows of the Society.

On the supposed possible Independence of Bessel's and Le Verrier's Expressions for the Mean Longitude of the Sun subject to the Conditions imposed upon them by their Use in practical Astronomy in the Determination of Mean Solar Time. By E. J. Stone, M.A., F.R.S.

The real point at issue between me and those who differ from my views with respect to the principal cause of the large existing discordances between Hansen's Tables of the Moon and observations, is as follows. Can we, consistently with the uses we have made of Bessel's and Le Verrier's expressions for the longitudes of the mean Sun, consider that there are two independent mean Suns without any necessary physical connection? That the question really resolves itself into this is easily shown. For let S denote the length of the sidereal day, and

$$T_1 = (1 + x_1)S, \quad T_2 = (1 + x_2)S,$$

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whatever values be assigned to x_1 or x_2. Then if ω_1, n_1, ω_2, n_2, are the angular velocities of the Earth about its axis, and the *true* Sun's mean motion in longitude in the units T_1 and T_2 respectively, we have identically

$$\frac{\omega_1}{\omega_2}=\frac{n_1}{n_2}=\frac{T_1}{T_2}.$$

This follows from the very definition of angular velocities and units of time, and therefore

$$\frac{T_1-T_2}{T_2}=\frac{n_1-n_2}{n_2}$$

which is my result; and one which must be true if n_1 and n_2 refer to any continuous motion.

But if we determine x_1 and x_2 so that

$$2\pi x_1=\alpha_1,\quad 2\pi x_2=\alpha_2,$$

α_1, α_2 being the circular measures of two angles; then we have, since 2π is the value of ω when S is the unit,

$$\omega_1=2\pi(1+x_1)=2\pi+\alpha_1$$
$$\omega_2=2\pi(1+x_2)=2\pi+\alpha_2.$$

$$\therefore\ \frac{\omega_1}{\omega_2}=\frac{T_1}{T_2}=\frac{2\pi+\alpha_1}{2\pi+\alpha_2}$$

$$\therefore\ \frac{T_1-T_2}{T_2}=\frac{\alpha_1-\alpha_2}{2\pi+\alpha_2}=\frac{\alpha_1-\alpha_2}{\omega_2}$$

$$\therefore\ \frac{T_1-T_2}{T_2}=\frac{\alpha_1-\alpha_2}{\omega_2}=\frac{n_1-n_2}{n_2}.$$

My result is therefore not inconsistent with, but identical with, that which has been previously obtained, for of course we can, if we please, assume

$$L_1=A_1+\alpha_1 t$$
$$L_2=A_2+\alpha_2 t,$$

and call L_1 and L_2 the longitudes of the mean Suns (1) and (2).

The real divergence between my views and those which have hitherto been accepted arises at this point; can we found our measures of mean solar time upon the expressions

$$L_1=A_1+\alpha_1 t,\ \text{and}\ L_2=A_2+\alpha_2 t,$$

and change from the one to the other, when they are used, as we do use them, without breaks in the continuity of our measures of time? or without the changes in the units which my theory indicates?

If this interchange of expressions be possible without any

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breaks of continuity in our measures of time or without equivalent changes in all our *adopted mean motions of the Moon and planets,* we can assume without error

$$\alpha_1 = \text{Bessel's adopted value,}$$

$$\alpha_2 = \text{Le Verrier's adopted value,}$$

and still infer that

$$\frac{T_1 - T_2}{T_2} = \frac{\alpha_1 - \alpha_2}{\omega_2} = \frac{n_B - n_{LV}}{\omega_2};$$

but if the two mean Suns cannot, consistently with the use which we have made of them, be regarded as independent, then it is quite impossible to have both $\alpha_1 = n_B$ and $\alpha_2 = n_{LV}$, as the discordance between my result and that usually accepted shows immediately.

Now, a conditional relationship between the two Suns is at once established when a *third* real continuous motion (that of the Moon, for instance) is connected with the motions of the centre of gravity of the Earth and that of the rotation of the Earth about its centre of gravity and expressed in terms of t. So long as we are dealing with the *two* independent motions, that of the centre of gravity of the Earth and that about the centre of gravity of the Earth, there is no absolute necessity of separating the concrete factors ω and t, and n and t, in the discussion of the ratio of the angular velocity of the Earth to that of the Sun's mean motion in longitude. But this separation has to be made before we can discuss the Moon's motion, or, indeed, our own motion with reference to its position in space. This is the point which, I believe, has been overlooked by mathematicians. It is one which might certainly not present itself naturally to a mind occupied only with the Solar Theory, and it presented itself to my mind simply because I was examining the Solar Theory with direct reference to the bearings of questions of time upon the Lunar Theory. Mathematically expressed, what I have been stating amounts to this.

If the Sun's mean longitude

$$= \epsilon + nt, \qquad (1)$$

and we find from our investigations of the Lunar Perturbations that the Moon's longitude

$$= A + Nt + \text{Periodic terms,} \qquad (2)$$

then N and n are necessarily referred to the *same* unit of time, *whatever that unit may be*; and the angle n, assigned or supposed to be known, is such that in *each unit of time* the Sun's mean longitude increases n. But before our Lunar Theory can be compared with the observations, we require the theoretical expressions for the observer's position as well as those for the centre of gravity of the Moon. As a matter of fact we do not, in our theoretical investigations, introduce any such thing as a

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mean Sun as distinguished from the true Sun, and we employ the same numerical value n, which expresses the Sun's mean motion in longitude, to establish the relationship between the motion of the centre of gravity of the Earth and the motion about that centre of gravity. The distinction, therefore, between the longitude of any mean Sun and the mean longitude of the true Sun is one not recognised in our theoretical work; and as we use, not an imaginary Sun, but the real Sun for establishing the relationship between the motion of the centre of gravity of the Earth and the motion about the centre of gravity of the Earth, it follows that if we do determine our mean time by equating the motion of the meridian to a mean Sun, then that mean Sun must be the true Sun in order that the times thus determined may be those upon which our theories are constructed, and for which they alone can be true.

It has been usual to eliminate the angular velocity of the Earth about its axis, from our equations, by supposing

$$\omega = 360 \times 60 \times 60'' + n. \qquad (3)$$

But the equations (1), (2), and (3) cannot co-exist except for *one* definite and determinate angle n; but if T be the unit of time in which the Sun's mean longitude does increase n, and the angle through which the Earth has turned about its axis increases $360 \times 60 \times 60'' + n$, we can at once find the true corresponding equations when we adopt any other angles n_1 or n_2 instead of n for the Sun's mean motion in longitude. For if T_1 and T_2 are two times expressed in terms of T such that $nT_1 = n_1$ and $nT_2 = n_2$: then if we adopt T_1 and T_2 as our units of time the angular velocities of the Sun in mean longitude will be n_1 and n_2 respectively; and conversely, if we adopt n_1 and n_2 for the Sun's mean motion in longitude, then the units of time must, consistently with the truth of (1) and (2), be T_1 and T_2.

If, therefore, t_1 and t_2 denote the times, when n_1 and n_2 are adopted, which correspond to the same *absolute instant* as that indicated by t when n is adopted, we have

$$\text{Sun's mean longitude} = \epsilon + nt = \epsilon + n_1 t_1 = \epsilon + n_2 t_2$$

$$\text{Moon's mean longitude} = A + Nt = A + N\frac{n_1}{n} . t_1 = A + N\frac{n_2}{n} . t_2$$

and

$$\begin{aligned}\text{R.A. of meridian in arc} &= \epsilon + \omega t = \epsilon + (360 \times 60 \times 60'' + n)t \\ &= \epsilon + (360 \times 60 \times 60'' + n_1)t_1 + 360 \times 60 \times 60''\left(\frac{n_1 - n}{n}\right) . t_1 \\ &= \epsilon + (360 \times 60 \times 60'' + n_2)t_2 + 360 \times 60 \times 60''\left(\frac{n_2 - n}{n}\right) . t_2\end{aligned}$$

Converting these expressions into time in the usual way we have

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$$\text{R.A. of meridian} = C + \left(24^h + \frac{n}{15}\right).t$$
$$= C + \left(24^h + \frac{n_1}{15}\right).t_1 + 24^h\left(\frac{n_1-n}{n}\right).t_1$$
$$= C + \left(24^h + \frac{n_2}{15}\right).t_2 + 24^h\left(\frac{n_2-n}{n}\right).t_2$$

Now, if n_1 and n_2 are the two values of the Sun's mean motion assumed by Bessel and Le Verrier, and B and V denote their adopted values of the sidereal times at corresponding mean noons, when $24^h.t_1 = 24^h \times$ some integer, and $24^h.t_2 = 24^h \times$ same integer, we shall have

$$\text{True R.A. of meridian at Bessel's mean noons} = B + 24^h\frac{n_1-n}{n}.t_1$$
$$\text{,, \quad ,, \quad Le Verrier's mean noons} = V + 24^h\frac{n_2-n}{n}.t_2$$

The difference, therefore, which arises from the use of Le Verrier's expression instead of Bessel's is

$$= V - B + 24^h.\frac{n_2-n_1}{n}.t.$$

The term

$$24^h.\frac{n_2-n_1}{n}.t$$

has been usually neglected on the impossible assumption that we could use either Bessel's or Le Verrier's expressions for the sidereal times at mean noon, without allowing for the change in the direction of the meridian between the two noons; and it has been the principal, if not the only argument against the acceptance of my views, that when you do neglect the terms

$$24^h.\frac{n_1-n}{n}.t_1 \quad \text{and} \quad 24^h.\frac{n_2-n}{n}.t_2,$$

which I have proved to be necessary corrections to B and V before the true mean times can be correctly deduced from the sidereal times, the times thus incorrectly found only differ by the 366th part of the difference indicated by my theory. If we suppose Le Verrier's value n_2 to be the true value, a supposition which will give the true difference, we have when the times were determined by B the true time

$$t_1 + \delta t_1 = t_1 + \frac{n_2-n_1}{n_2}.t_1,$$

and if we determine, therefore, the Moon's mean motion with these defective times t_1 we shall find instead of N, the value $N+\delta N$ such that

$$N\left(t_1 + \frac{n_2-n_1}{n_2}.t_1\right) = (N+\delta N)t_1$$

or

$$\delta N = \frac{n_2-n_1}{n_2}.N,$$

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and consequently, in 1864, when we replaced B by V, we ought to have changed the adopted value of the Moon's motion to the extent indicated; and all our other adopted mean motions should have been similarly changed to allow for the change in the adopted unit of time.

It is difficult to understand how any practical astronomer can seriously maintain that we are in these questions of mean solar time only concerned with the Sun's hour-angles, and not with the direction of the meridian in space. As a matter of fact our clock-errors are deduced from sidereal times which directly depend upon the position of the meridian with respect to the equinox.

The whole of our difficulties have arisen from attempts to carry on simultaneously an approximation to the Sun's mean motion in the true mean solar day, and to use that true mean solar day as our unit of time.

The equation $\omega T - nT = 2\pi$ has been theoretically treated as if the factor T, the adopted unit of time, did not exist, and as if the true equation was $\omega - n = 2\pi$. It has therefore been thought that the only possible way of keeping $\delta(\omega T) - \delta(nT) = 0$ was to take $\delta\omega = \delta n$, a condition which keeps the Sun's hour-angle unchanged, but throws the direction of the meridian of the observer out of its true position; such a solution cannot therefore be the true solution of our physical problem. Directly the unit T is introduced the difficulty disappears. All we require to keep the hour-angle unchanged is that

$$\delta\omega T + \omega\delta T = \delta n T + n\delta T,$$

which is satisfied if

$$-\frac{\delta T}{T} = \frac{\delta\omega}{\omega} = \frac{\delta n}{n},$$

and as these relations satisfy the necessary conditions that the Sun's mean longitude, and the direction of the meridian in space, should both be unaltered by the changes introduced into the adopted value of n, the solution thus obtained is the only possible solution which will bear comparison with the facts of observation.

Although it may appear useless to continue researches of which the fundamental principle is still disputed by astronomers of note, I may mention that, besides accounting for the missing 4″ required to reconcile our observations with the theoretical value of the secular acceleration of the Moon's mean motion in longitude, I have convinced myself that the long inequality with a coefficient of 21″ introduced by Hansen, but whose coefficient was found to be insensible by Delaunay, is, to very nearly its full extent at all events, an empirical inequality rendered necessary to reconcile theory and observation on account of breaks in the continuity of our measures of time similar to that which took place in 1864. I have not seen at present any evidence which weakens in the slightest degree my confidence in the accuracy of my results.